UNIMORE
UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

Dipartimento di Ingegneria "Enzo Ferrari"

MILLE CHILI

Progettazione Assistitia di Organi di Macchine

Sara Mantovani
sara.mantovani@unimore.it

Agenda

Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- Problem definition and nomenclature
- Reaction forces evaluation
- Skew-symmetry BCs
- Castigliano's theorem general formulation
- Deflection of the structure
- Torsional stiffness of a space frame
- Further considerations
- References

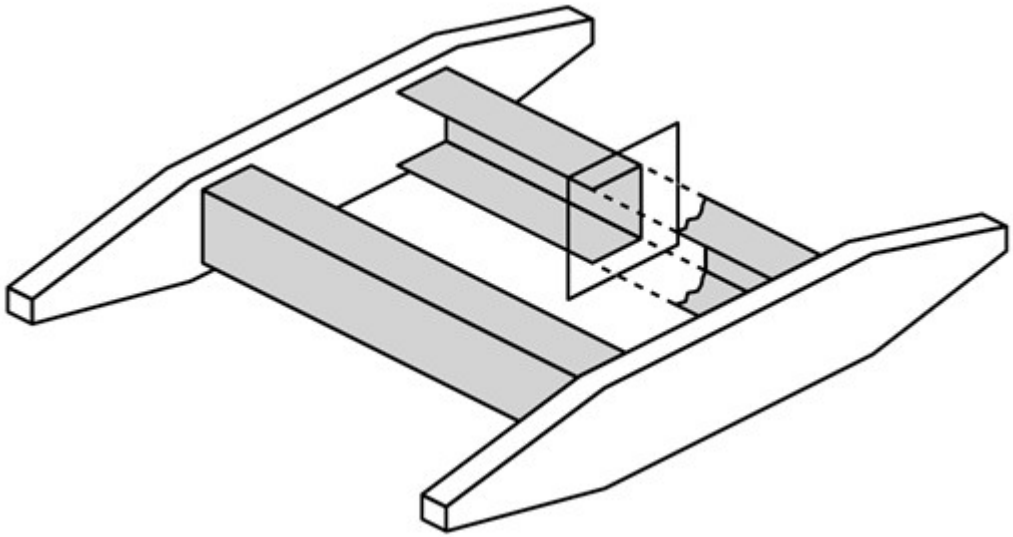
Agenda

Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- **Problem definition and nomenclature**
- **Reaction forces evaluation**
- Skew-symmetry BCs
- Castigliano's theorem general formulation
- Deflection of the structure
- Torsional stiffness of a space frame
- Further considerations
- References

Castigliano's Theorem

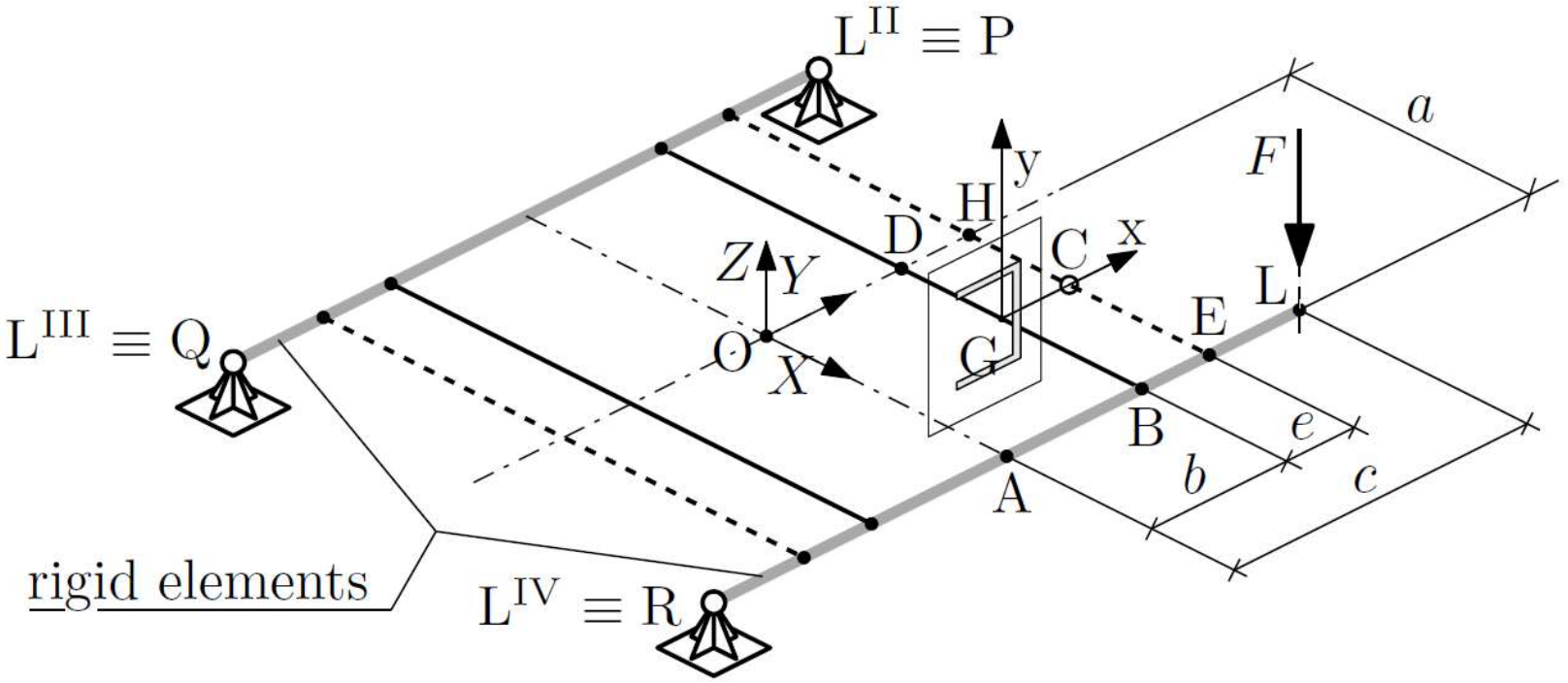
CASE E: Ladder Frame – Problem Introduction



Evaluate the torsional stiffness (t_s) of the chassis supported at three wheel centers and loaded at the fourth adopting Castigliano's theorem.

Castigliano's Theorem

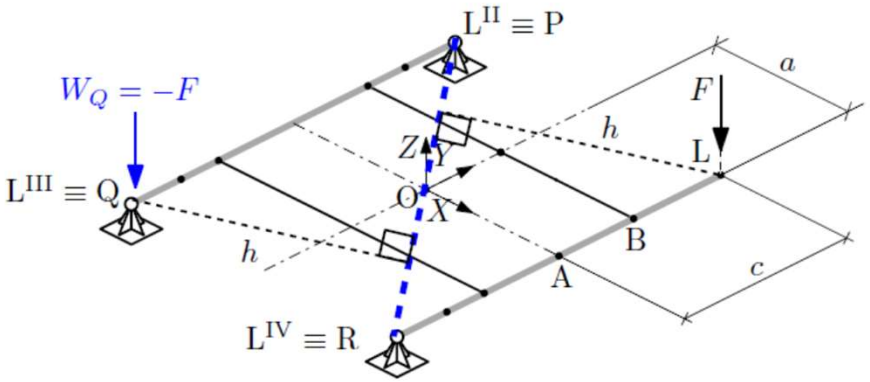
CASE E: Ladder Frame – Nomenclature

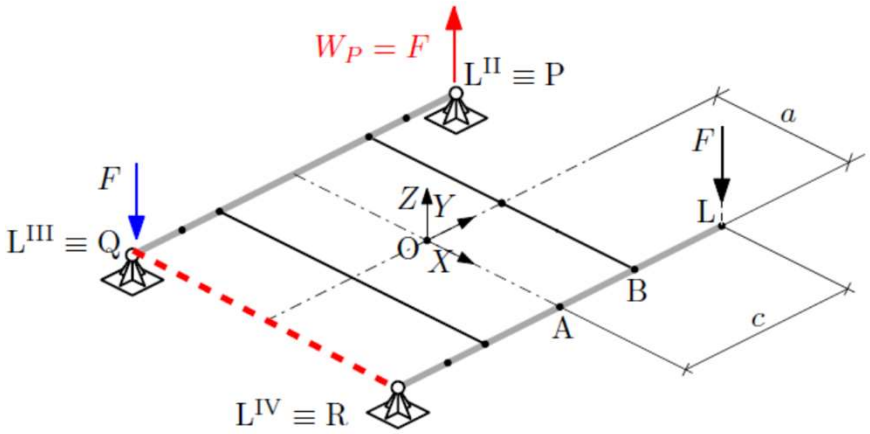


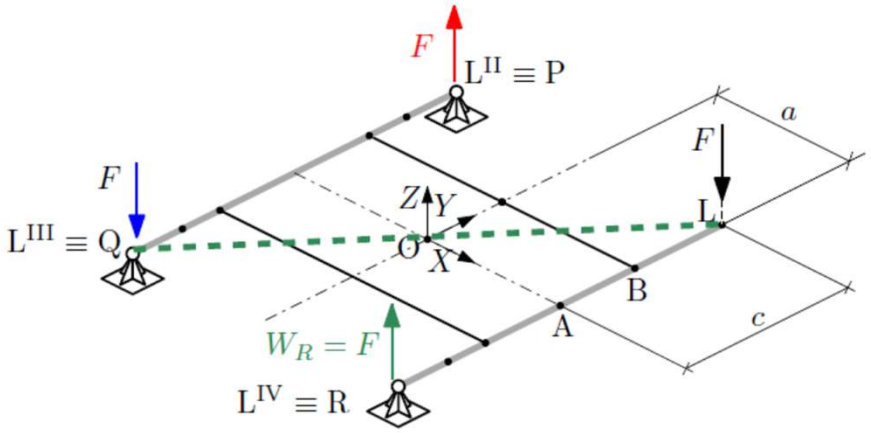
O, X, Y, Z: global coordinate system
x, y, z: local system applied at the beam cross-section centre of gravity (*G*)
a, b, c, e: dimensions of the frame
F: external force
C: shear centre of the beam cross-section
A, B, D, E, H, L, P, Q, R: crucial points of the frame

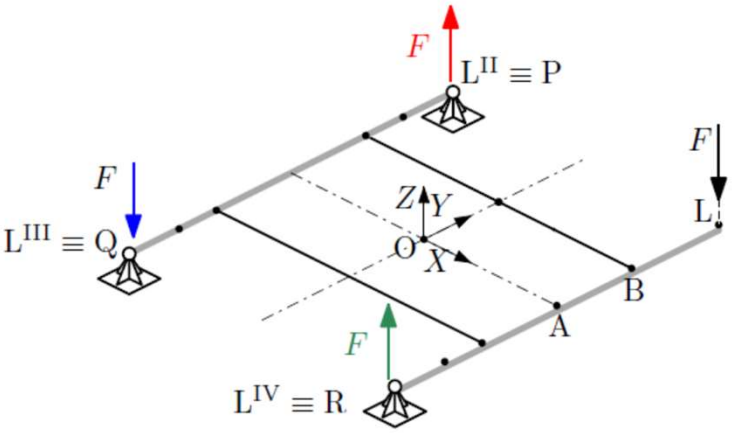
Ladder Frame

Reaction forces evaluation









Agenda

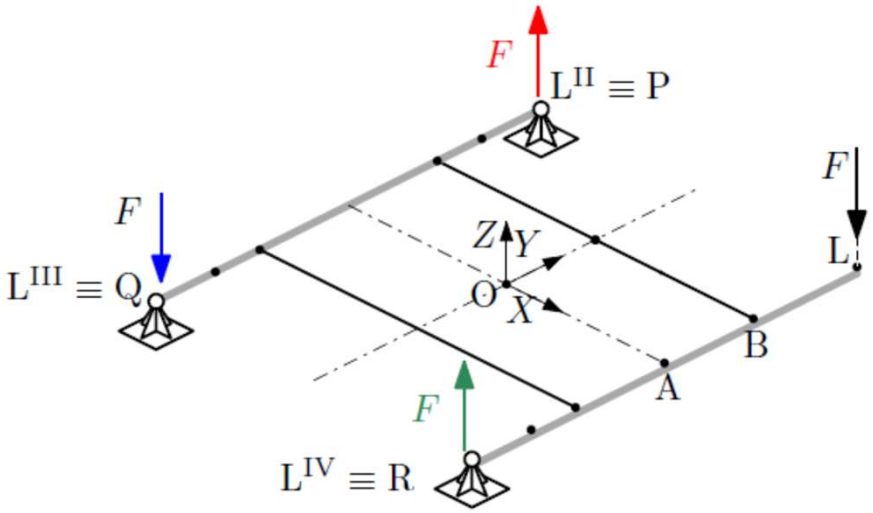
Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- Problem definition and nomenclature
- Reaction forces evaluation
- **Skew-symmetry BCs**
- Castigliano's theorem general formulation
- Deflection of the structure
- Torsional stiffness of a space frame
- Further considerations
- References

Ladder Frame

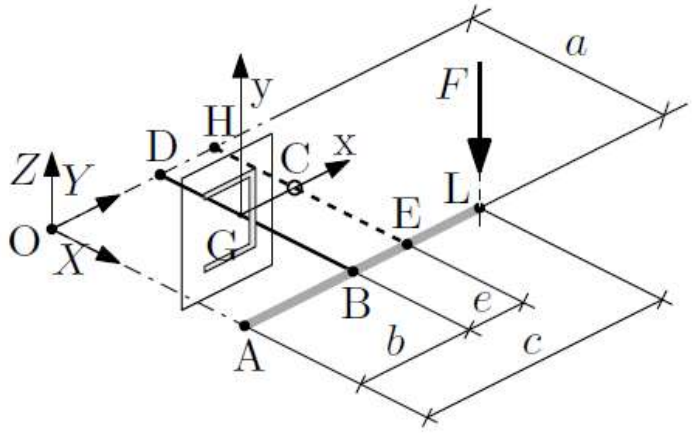
Skew-symmetry definition

Complete – ladder structure



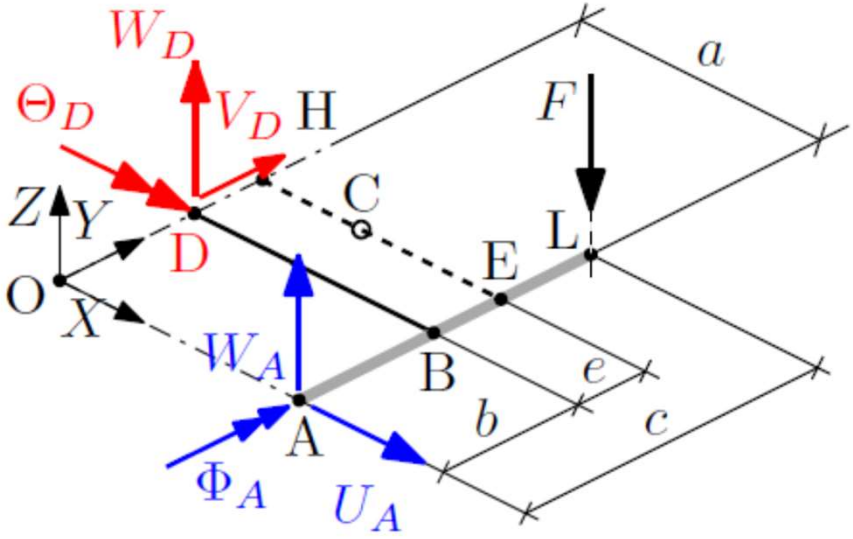
Quarter – ladder structure

We decide to neglect the *XY* skew- symm plane to reduce the problem complexity.



Ladder Frame

Skew-symmetry definition

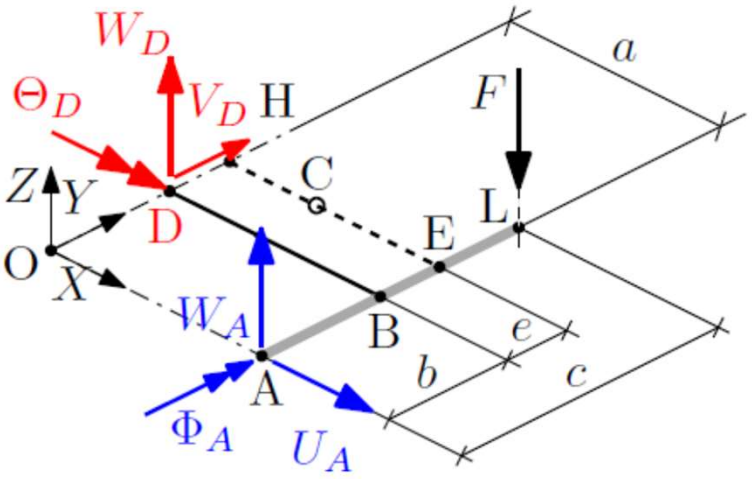


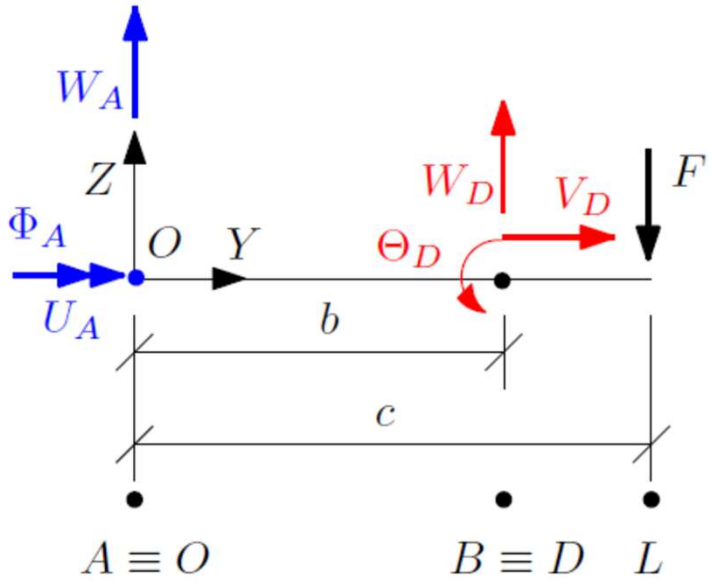
Quarter – ladder structure
Skew-symmetry BCs

Nomenclature:
U_i, V_i, W_i: Reaction forces related to *x, y, z* local axes respectively.
Θ_i, Φ_i, Ψ_i: Reaction moments related to *x, y, z* local axes respectively.

Ladder Frame

Skew-symmetry definition





YZ plane

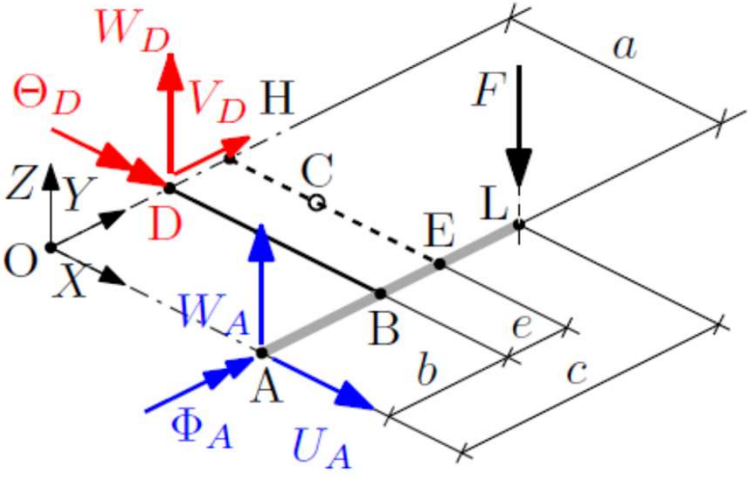
X: outward

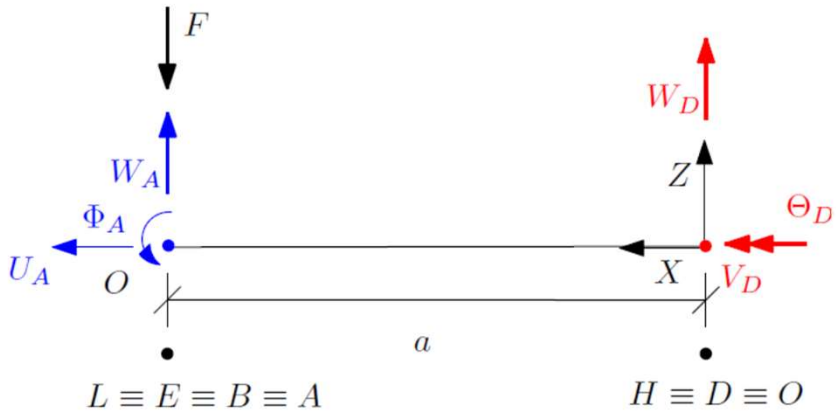
Quarter – ladder structure

Skew-symmetry BCs

Ladder Frame

Skew-symmetry definition



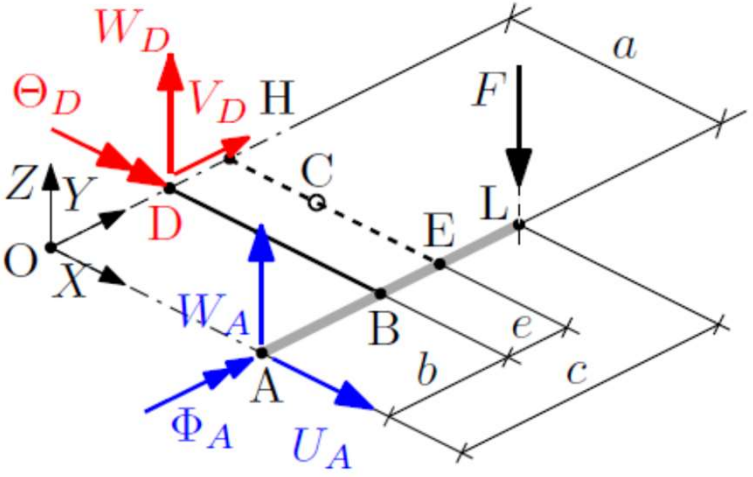


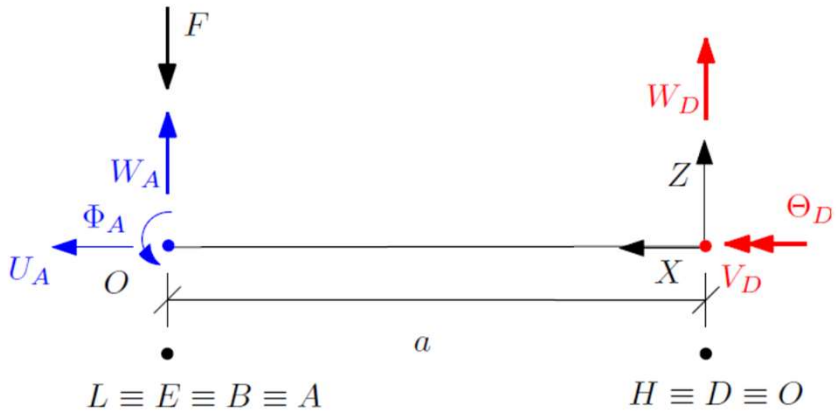
Quarter – ladder structure
Skew-symmetry BCs

XZ plane
Y: outward

Ladder Frame

Skew-symmetry definition





Quarter – ladder structure
Skew-symmetry BCs

XZ plane
Y: outward

Agenda

Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- Problem definition and nomenclature
- Reaction forces evaluation
- Skew-symmetry BCs
- **Castigliano's theorem general formulation**
- **Deflection of the structure**
- Torsional stiffness of a space frame
- Further considerations
- References

Ladder Frame

Castigliano's Theorem general formulation

The linear density of the elastic potential (alternatively named internal) energy for the spatial rectilinear beam may be derived as a function of its cross section resultants, namely

$$\frac{dU}{dl} = \frac{J_{\eta\eta}M_\xi^2 + J_{\xi\xi}M_\eta^2 + 2J_{\xi\eta}M_\xi M_\eta}{2E\left(J_{\xi\xi}J_{\eta\eta} - J_{\xi\eta}^2\right)} + \frac{N^2}{2EA} \tag{1.37}$$

$$+ \frac{\chi_\xi S_\xi^2 + \chi_\eta S_\eta^2 + \chi_{\xi\eta}S_\eta S_\xi}{2GA} + \frac{M_t^2}{2GK_t} \tag{1.38}$$

where

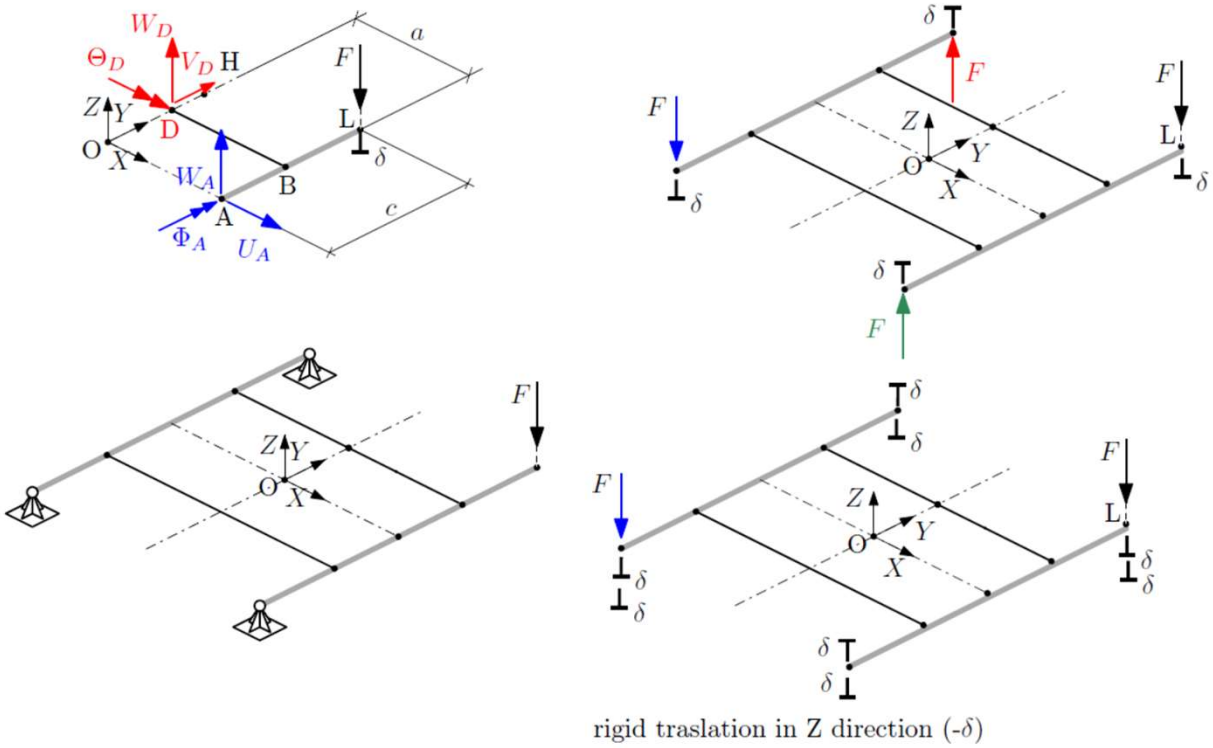
- A, $J_{\eta\eta}$, $J_{\xi\xi}$ and $J_{\xi\eta}$ are the section area and moments of inertia, respectively;
- K_t is the section torsional stiffness (**not** generally equivalent to its polar moment of inertia);
- E and G are the material Young Modulus and Shear Modulus, respectively; the material is assumed homogeneous, isotropic and linearly elastic.

The shear energy normalized coefficients χ_η,χ_ξ,$\chi_{\xi\eta}$ are specific to the cross section geometry, and may be collected from the expression of the actual shear strain energy due to concurrent action of the S_η, S_ξ shear forces.

Ladder Frame

Deflection of the structure

The internal energy *U* calculated for the quarter-ladder frame allows the evaluation of the deflection of the frame that is ¼ of the deflection of the overall structure.
It might be seen this aspect by the congruency garanteed at the support of the complete structure.

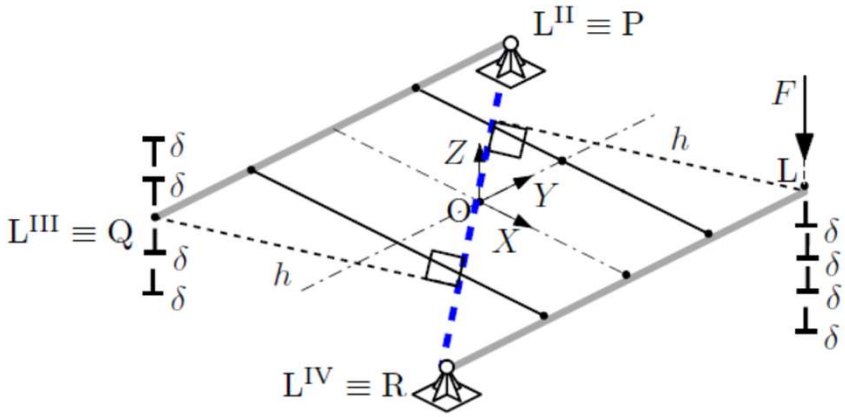


rigid traslation in Z direction (-δ)

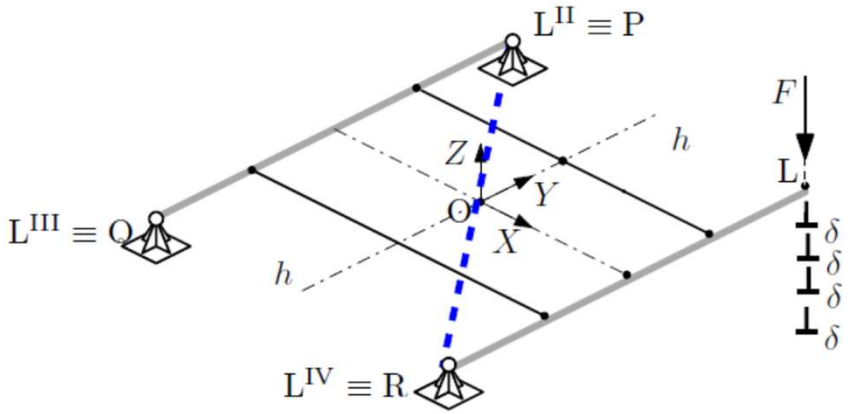
Ladder Frame

Deflection of the structure

The internal energy *U* calculated for the quarter-ladder frame allows the evaluation of the deflection of the frame that is ¼ of the deflection of the overall structure.
By the kinematic concompatibility reinstated (three-supports), the calcultation of the oveall dflection is evaluated.



eq. rotation along the PR-axis
The Q point must be increased on direction Z of $(2\dot{\delta})$



Ladder Frame

Deflection of the structure

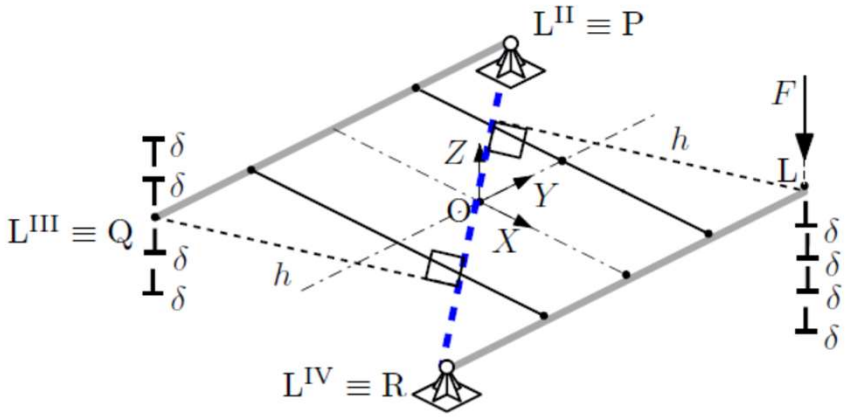
t_s : torsional stiffness [Nm/deg]
M_t: torsional moment
$\Delta\theta$: rotation

$$t_s = \frac{M_t}{\Delta\theta} = \frac{2Fc}{\Delta\theta}$$

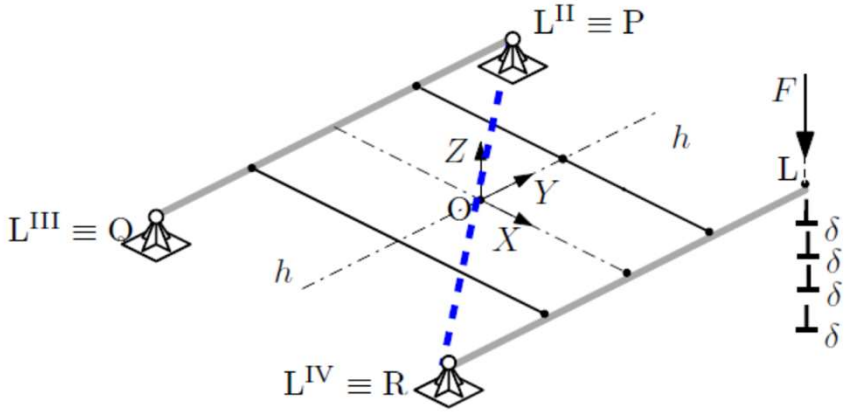
$$\Delta\theta = \frac{4\delta}{2c}$$

where

$$t_s = \frac{M_t}{\Delta\theta} = \frac{F(2c)^2}{4\delta}$$



eq. rotation along the PR-axis
The Q point must be increased on direction Z of $(2\dot{\delta})$



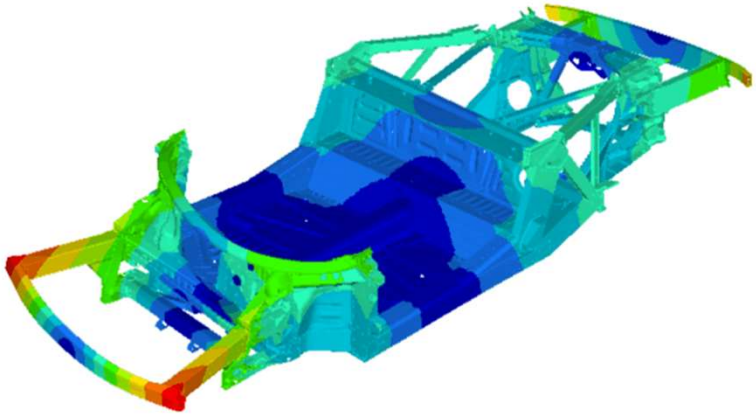
Agenda

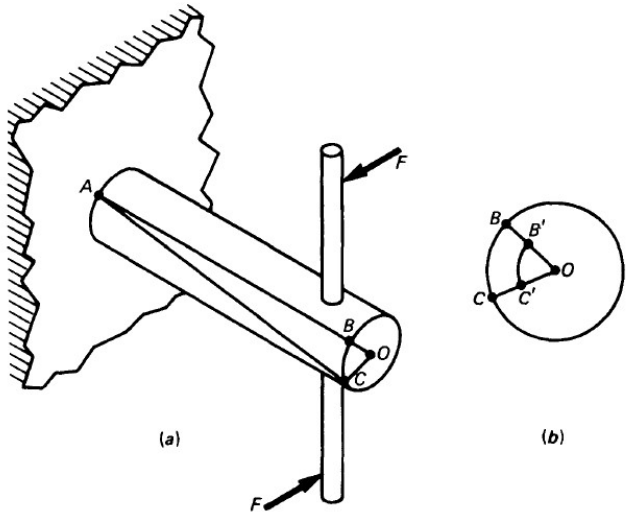
Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- Problem definition and nomenclature
- Reaction forces evaluation
- Skew-symmetry BCs
- Castigliano's theorem general formulation
- Deflection of the structure
- **Torsional stiffness of a space frame**
- **Further considerations**
- References

Torsional stiffness

Space-frame

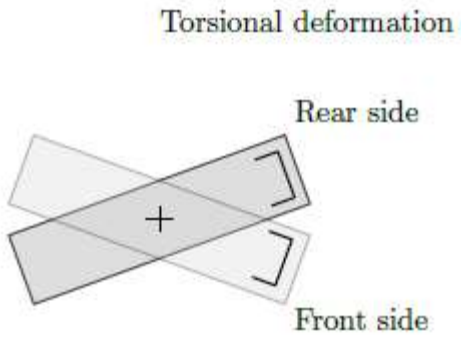




$E_t = t_s * P * C / (M_{BIW} * 1000)$

- E_t: torsional efficiency [$m^4/(s^2\ deg)$]
- t_s : torsional stiffness [Nm/deg] = M_t/θ
- P: wheelbase [m]
- C: axle track [m]
- M: weight [kg]

for a *Body in White* frame structure.

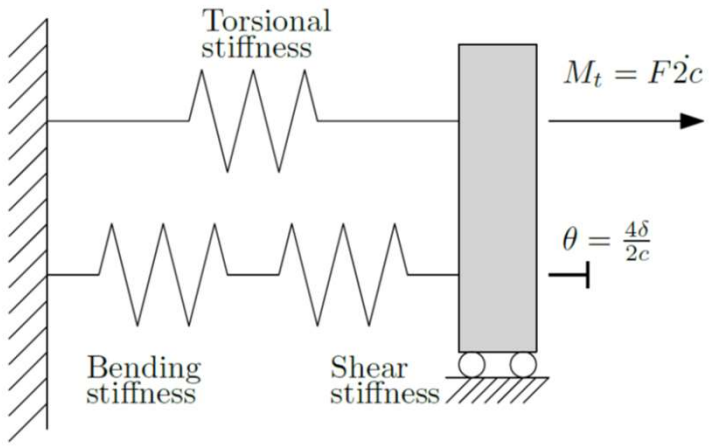


Ladder Frame

Further considerations

summarizing the results, we obtain the following table

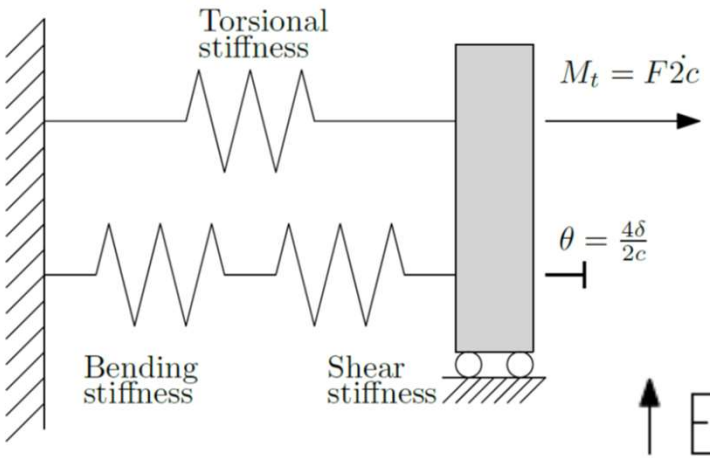
trs	bnd	shr	residual stiffness
1	1	1	100. %
1	1	0	7.2%
1	0	1	7.2%
1	0	0	7.2%
0	1	1	92.8%
0	1	0	0.0%
0	0	1	0.0%
0	0	0	0.0%

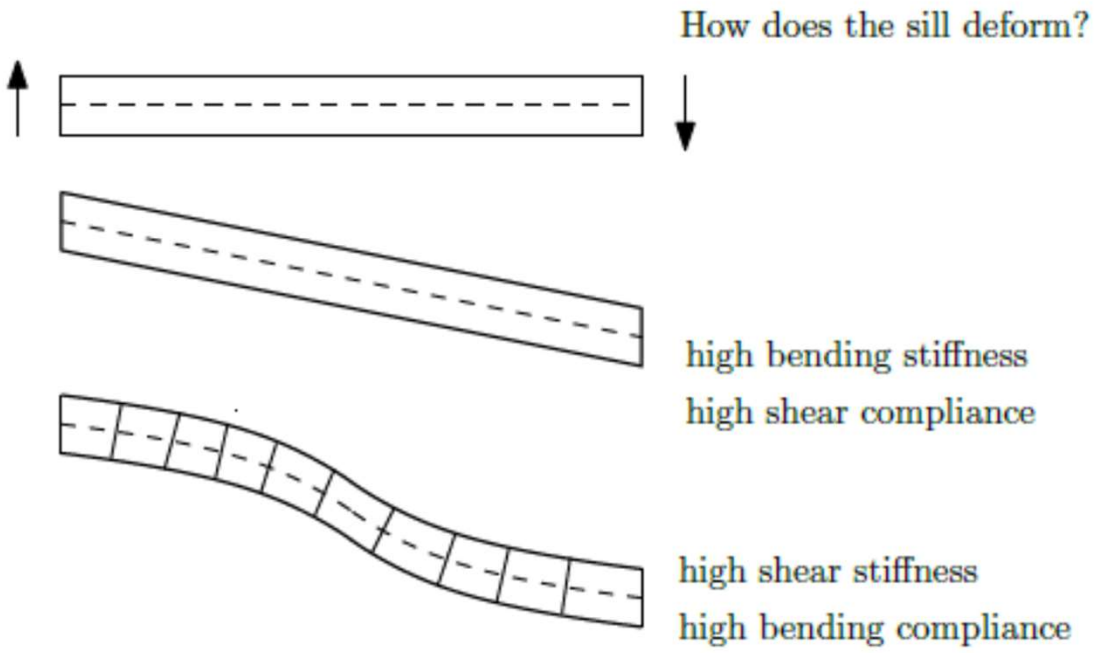


which may be rationalized according to the following scheme

Ladder Frame

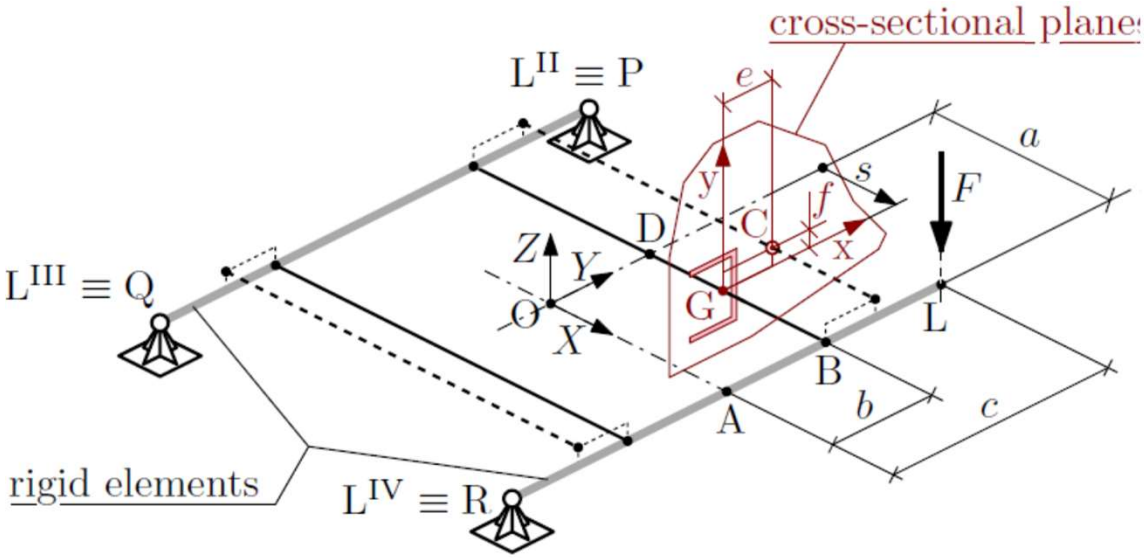
Further considerations

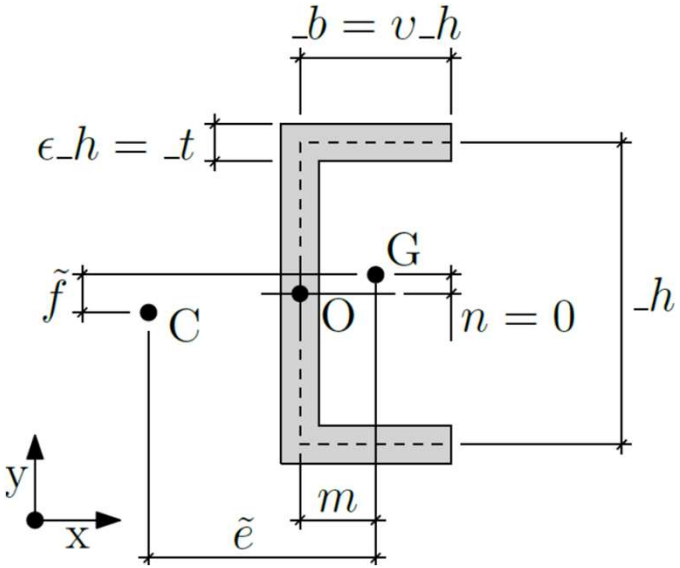




Ladder Frame

Further considerations





Generic sections (without plane of symmetry).

Agenda

Castigliano's Theorem analysis of a ladder frame (Case E) by Maxima

- Problem definition and nomenclature
- Reaction forces evaluation
- Skew-symmetry BCs
- Castigliano's theorem general formulation
- Deflection of the structure
- Torsional stiffness of a space frame
- Further considerations
- **References**

References

LAB Maxima files saved as:
`quarter_ladder_frame_v005_BASE2019.wxmx`
`quarter_ladder_frame_v005_BASE2019_rev02.wxmx`

Torsional stiffness testing:
http://www.optimumg.com/technical/torsional-stiffness-2/
of a ladder frame vehicle at 1:30 minutes of the video
https://www.youtube.com/watch?v=opNSP59TTcY
of different panels varying the material under investigation

«Delle cose che leggi qui alcune sono buone, altre mediocri, la maggior parte cattive. Così e non altrimenti, Avito, son fatti i libri».

Marziale, Epigrammi

Sara Mantovani
Via Vivarelli, 10
41125, Modena, Italy
Mail: sara.mantovani@unimore.it
Mail: millechili@unimore.it
Phone: +39 059 2056280