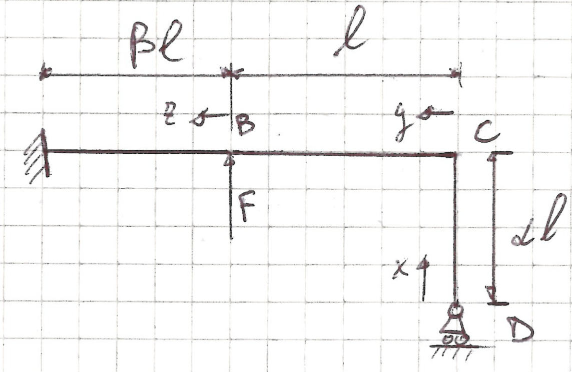
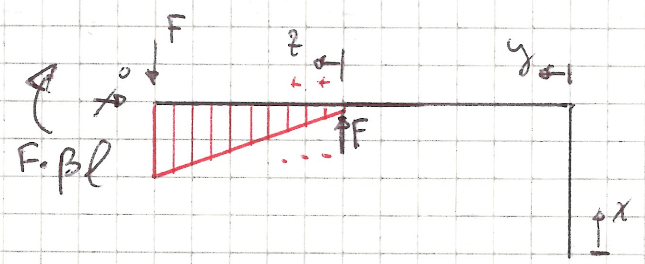
## Esercizio 2.17.

Si risolva la seguente struttura staticamente indeterminata con il teorema di Castigliano:




Considero la struttura principale caricata dalla sola forza F:

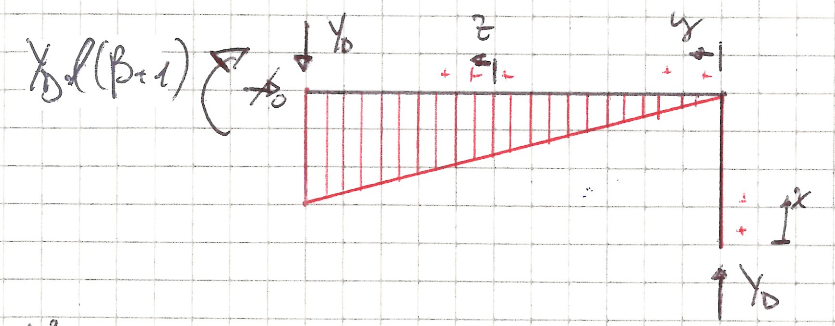



$Mf_F(x) = 0$

$Mf_F(y) = 0$

$Mf_F(z) = -F \cdot z$

Considero ora la stessa struttura principale caricata ora dalla sola reazione $Y_D$.




$Mf_{Y_D}(x) = 0$

$Mf_{Y_D}(y) = -Y_D \cdot y$

$Mf_{Y_D}(z) = -Y_D \cdot l - Y_D \cdot z$

Applico ora il Teorema di Castigliano.

$$U = \frac{1}{2EJ}\left[\int_0^{\alpha l}\left(M_{f_F}(x) + \cancel{M_{f_{Y_D}}(x)}\right)^2 dx + \int_0^{l}\left(\cancel{M_{f_F}(y)} + M_{f_{Y_D}}(y)\right)^2 dy + \int_0^{\beta l}\left(M_{f_F}(z) + M_{f_{Y_D}}(z)\right)^2 dz\right] =$$

$$= \frac{1}{2EJ}\left[\int_0^{l}\left(-Y_D \cdot y\right)^2 dy + \int_0^{\beta l}\left(-F \cdot z - Y_D \cdot l - Y_D \cdot z\right)^2 dz\right] =$$

$$= \frac{1}{2EJ}\left[Y_D^2 \cdot \frac{l^3}{3} + \int_0^{\beta l}\left(F^2 z^2 + Y_D^2 l^2 + Y_D^2 z^2 + 2FY_D z \cdot l + 2FY_D z^2 + 2Y_D^2 l z\right) dz\right]$$

$$= \frac{1}{2EJ} \cdot \left(Y_D^2 \frac{l^3}{3} + F^2 \frac{\beta^3 l^3}{3} + Y_D^2 \frac{\beta l^3}{1} + Y_D^2 \frac{\beta^3 l^3}{3} + \cancel{2} F Y_D \cdot \frac{\beta^2 l^3}{\cancel{2}} + 2FY_D \frac{\beta^3 l^3}{3} + \cancel{2} Y_D^2 \frac{\beta^2 l^3}{\cancel{2}}\right)$$

$$\frac{\partial U}{\partial Y_D} = \delta_D = 0 = \frac{1}{2EJ}\left(2Y_D \frac{l^3}{3} + 0 + 2Y_D \beta l^3 + 2Y_D \frac{\beta^3 l^3}{3} + F \cdot \beta^2 l^3 + \frac{2}{3} F \beta^3 l^3 + 2Y_D \frac{\beta^2 l^3}{1}\right)$$

c'è il vincolo carrello

$$0 = \frac{2}{3} Y_D + 2Y_D \cdot \beta + \frac{2}{3} Y_D \cdot \beta^3 + F \cdot \beta^2 + \frac{2}{3} F \cdot \beta^3 + 2Y_D \cdot \beta^2$$

$$Y_D\left(\frac{2}{\cancel{3}} + \frac{6}{\cancel{3}}\beta + \frac{2}{\cancel{3}}\beta^3 + \frac{6}{\cancel{3}}\beta^2\right) = -F\left(\beta^2 \cdot \frac{3}{\cancel{3}} + \frac{2}{\cancel{3}}\beta^3\right)$$

$$Y_D = F \cdot (-1) \cdot \frac{3\beta^2 + 2\beta^3}{2 + 6\beta + 6\beta^2 + 2\beta^3}$$