$\underline{\underline{b}}$, which is in general nonzero, vanishes if the material is symmetrically distributed with respect to the reference surface.

In the commonwise case of tt homogeneous material, and null offset[13] we have

$$\underline{\underline{a}} = h\,\underline{\underline{D}} \qquad\qquad \underline{\underline{b}} = \underline{\underline{0}} \qquad\qquad \underline{\underline{c}} = \frac{h^3}{12}\,\underline{\underline{D}}\,,$$

i.e. the membrane stiffness varies linearly with the wall thickness, the flexural stiffness varies with the cube of the thickness, and the membrane and the flexural loadings are mutually uncoupled. Such a laminate elastic properties dependence on thickness essentially holds also for laminates, if the tt distribution of the various materials is kept comparable.

2.1.5 The transverse shear stress/strain components

A full treatise on the title topic is, due to its complexity, bspc; starting points for further investigation my be found in [3], [4] or in the theory manual of your favourite fe solver[14].

The two transverse shear components

$$\underline{\gamma}_z = \begin{bmatrix} \bar{\gamma}_{yz} \\ \bar{\gamma}_{zx} \end{bmatrix}$$

are in fact more directly recognizable as further contributions to the $\left(\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right)$ normal deflection gradient, with respect to what is attributable to flexure alone, than tt averages of actual, pointwise shear strains – see e.g. Figure 2.1.

Since no direct procedure is available[15] for directly probing the $\bar{\gamma}_{yz}$ and $\bar{\gamma}_{zx}$ quantities in a deflected plate, their definition is inevitably nebulous.

The two transverse shear stress resultants defined in Eq. 2.17

$$\underline{q}_z = \begin{bmatrix} q_{xz} \\ q_{yz} \end{bmatrix}$$

[13]In the presence of a nonzero offset between the reference and the median planes, the uncoupled nature of the plate membrane/flexural loadings is only *formally* lost. If the same problem is considered based on a median reference plane, in fact, such a property is obviously restored.

[14]See e.g. MSC.Marc 2013.1 Documentation, Vol. A, pp. 433-436

[15]as far as the writer knows

are assumed to perform work[16] on the same $\bar{\gamma}_{yz}$ and $\bar{\gamma}_{zx}$ transverse shear components, respectively; the transverse shear contribution to the elastic strain energy per unit ref. surface area is hence

$$\upsilon^{\ddagger} = \frac{1}{2}\underline{\gamma}_{z}^{\top}\underline{q}_{z} = \frac{1}{2}\bar{\gamma}_{xz}q_{xz} + \frac{1}{2}\bar{\gamma}_{yz}q_{yz}. \tag{2.22}$$

The constitutive equation for the transverse shear is set at normal segment (vs. punctual) level, with the declared aim of collecting the elastic strain energy contributions along the thickness, and they are usually formulated as

$$\upsilon^{\ddagger} = \frac{1}{2}\underline{\gamma}_{z}^{\top}\underbrace{\left[\chi\left(\frac{1}{h}\int_{h}\underline{\underline{G}}^{-1}dz\right)^{-1}h\right]}_{\underline{\underline{\Gamma}}}\underline{\gamma}_{z} \tag{2.23}$$

where $\underline{\underline{G}}$ is the pointwise constitutive matrix for the transverse shear components[17] – which is considered in terms of its tt harmonic average[18], χ is a *shear correction factor* – which accommodates for possibly any incongruence in the formulation, and $\underline{\underline{\Gamma}}$ is an emended transverse shear constitutive matrix for the whole plate.

In the case of isotropic materials, $\underline{\underline{G}}$ is a diagonal matrix whose terms equate the shear modulus, i.e.

$$\underline{\underline{G}} = \frac{E}{2\,(1+\nu)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

whereas the χ shear correction factor is usually assumed as $\frac{5}{6}$ if the material is tt uniform[19]; different χ values are however proposed in literature, see e.g. [5], along with different procedures[20] for evaluating

[16] in particular, work for unit reference surface area

[17] $\underline{\underline{G}}$ is the 2 by 2 matrix s.t., pointwisely, $\begin{bmatrix} \tau_{zx} \\ \tau_{yz} \end{bmatrix} = \underline{\underline{G}}\begin{bmatrix} \gamma_{zx} \\ \gamma_{yz} \end{bmatrix}$.

[18] the shear sliding is accumulated across the various layers, rather than the elastic reactions, thus assuming an *in series* layout of equivalent springs

[19] please note the parallel with the inverse 1.2 correction factor for the shear contribution to the beam elastic strain energy, proper of the solid rectangular cross section.

[20] we report as an example the notable case of of honeycomb panels – whose transverse shear compliance is rarely negligible, in which Γ is defined as the $\underline{\underline{G}}_{\text{foam}}$ transverse shear constitutive matrix for the foam/honeycomb material interposed between the outer skins, multiplied by the overall panel thickness h; in this case the χ transverse shear correction factor is implicitly defined as unity.

$\underline{\underline{\Gamma}}$. By comparing Eqns. 2.22 and 2.23 we also derive the *de facto* transverse shear constitutive relation

$$\underline{q}_z = \underline{\underline{\Gamma}}\, \underline{\gamma}_z. \tag{2.24}$$

for the Mindlin shear deformable plate.

In the case pointwise values are requested for the τ_{zx} and τ_{yz} stress components – e.g. in the analysis of interlaminar stresses in composite laminates, those quantities are derived from the assumed absence of shear stresses on the lower surface, and by accumulating the ip stress component contributions to the x and y translational equilibria up to the desired z sampling height. We hence obtain

$$\tau_{zx}(z) = -\int_{-\frac{h}{2}+o}^{z} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} dz = \int_{z}^{+o+\frac{h}{2}} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} dz \tag{2.25}$$

$$\tau_{yz}(z) = -\int_{-\frac{h}{2}+o}^{z} \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} dz = \int_{z}^{+o+\frac{h}{2}} \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} dz. \tag{2.26}$$

The parallel is evident with the Jourawsky theory of shear for beams.

2.1.6 Hooke's law for the orthotropic lamina

Hooke's law for the orthotropic material ip stress conditions, with respect to principal axes of orthotropy;

$$\underline{\underline{D}}_{123} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{2.27}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \underline{\underline{T}}_1 \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \qquad \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \underline{\underline{T}}_2 \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{2.28}$$

where

$$\underline{\underline{T}}_1 = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{2.29}$$

$$\underline{\underline{T}}_2 = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \tag{2.30}$$