DAMPING CROSS-REFERENCE

There are at least eleven parameters commonly used to express damping. Cross-reference formulas are given in Tables 1A through 1C. The formulas are taken from Reference 1.

Let ω_n be the natural frequency in units of radians per second. Note that $\omega_n = 2\pi f_n$, where f_n is in units of Hertz.

Table 1A. Damping Reference				
Parameter	3 dB Bandwidth $\Delta\omega$ (rad/sec)	3 dB Bandwidth Δf (Hz)	Damping Frequency fd (Hz)	Loss Factor η
3 dB Bandwidth $\Delta\omega$ (rad/sec)	–	$\Delta\omega = 2\pi\Delta f$	$\Delta\omega = 4\pi fd$	$\Delta\omega = \omega_n \eta$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{\Delta\omega}{2\pi}$	–	$\Delta f = 2fd$	$\Delta f = \frac{\omega_n \eta}{2\pi}$
Damping Frequency fd (Hz)	$fd = \frac{\Delta\omega}{4\pi}$	$fd = \frac{\Delta f}{2}$	–	$fd = \frac{\omega_n \eta}{4\pi}$
Loss Factor η	$\eta = \frac{\Delta\omega}{\omega_n}$	$\eta = \frac{2\pi\Delta f}{\omega_n}$	$\eta = \frac{4\pi fd}{\omega_n}$	–
Fraction of Critical Damping ζ	$\zeta = \frac{\Delta\omega}{2\omega_n}$	$\zeta = \frac{\pi\Delta f}{\omega_n}$	$\zeta = \frac{2\pi fd}{\omega_n}$	$\zeta = \frac{\eta}{2}$
Quality Factor Q	$Q = \frac{\omega_n}{\Delta\omega}$	$Q = \frac{\omega_n}{2\pi\Delta f}$	$Q = \frac{\omega_n}{4\pi fd}$	$Q = \frac{1}{\eta}$
Decay Constant σ (1/sec)	$\sigma = \frac{\Delta\omega}{2}$	$\sigma = \pi\Delta f$	$\sigma = 2\pi fd$	$\sigma = \frac{\omega_n \eta}{2}$
Time Constant τ (sec)	$\tau = \frac{2}{\Delta\omega}$	$\tau = \frac{1}{\pi\Delta f}$	$\tau = \frac{1}{2\pi fd}$	$\tau = \frac{2}{\omega_n \eta}$
Reverberation Time RT_{60} (sec)	$RT_{60} = \frac{13.8}{\Delta\omega}$	$RT_{60} = \frac{2.2}{\Delta f}$	$RT_{60} = \frac{1.1}{fd}$	$RT_{60} = \frac{13.8}{\omega_n \eta}$
Decay Rate D (dB/sec)	$D = 4.34\Delta\omega$	$D = 27.3\Delta f$	$D = 54.6 fd$	$D = 4.34\omega_n \eta$
Logarithmic Decrement δ	$\delta = \frac{\pi\Delta\omega}{\omega_n}$	$\delta = \frac{2\pi^2 \Delta f}{\omega_n}$	$\delta = \frac{4\pi^2 fd}{\omega_n}$	$\delta = \pi\eta$

Table 1B. Damping Reference				
Parameter	Fraction of Critical Damping ζ	Quality Factor Q	Decay Constant σ (1/sec)	Time Constant τ (sec)
3 dB Bandwidth $\Delta\omega$ (rad/sec)	$\Delta\omega = 2\omega_n \zeta$	$\Delta\omega = \frac{\omega_n}{Q}$	$\Delta\omega = 2\sigma$	$\Delta\omega = \frac{2}{\tau}$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{\omega_n \zeta}{\pi}$	$\Delta f = \frac{\omega_n}{2\pi Q}$	$\Delta f = \frac{\sigma}{\pi}$	$\Delta f = \frac{1}{\pi\tau}$
Damping Frequency fd (Hz)	$fd = \frac{\omega_n \zeta}{2\pi}$	$fd = \frac{\omega_n}{4\pi Q}$	$fd = \frac{\sigma}{2\pi}$	$fd = \frac{1}{2\pi\tau}$
Loss Factor η	$\eta = 2\zeta$	$\eta = \frac{1}{Q}$	$\eta = \frac{2\sigma}{\omega_n}$	$\eta = \frac{2}{\omega_n \tau}$
Fraction of Critical Damping ζ	–	$\zeta = \frac{1}{2Q}$	$\zeta = \frac{\sigma}{\omega_n}$	$\zeta = \frac{1}{\omega_n \tau}$
Quality Factor Q	$Q = \frac{1}{2\zeta}$	–	$Q = \frac{\omega_n}{2\sigma}$	$Q = \frac{\omega_n \tau}{2}$
Decay Constant σ (1/sec)	$\sigma = \omega_n \zeta$	$\sigma = \frac{\omega_n}{2Q}$	–	$\sigma = \frac{1}{\tau}$
Time Constant τ (sec)	$\tau = \frac{1}{\omega_n \zeta}$	$\tau = \frac{2Q}{\omega_n}$	$\tau = \frac{1}{\sigma}$	–
Reverberation Time RT_{60} (sec)	$RT_{60} = \frac{6.9}{\omega_n \zeta}$	$RT_{60} = \frac{13.8Q}{\omega_n}$	$RT_{60} = \frac{6.9}{2\sigma}$	$RT_{60} = 6.9\tau$
Decay Rate D (dB/sec)	$D = 8.68\omega_n \zeta$	$D = \frac{4.34\omega_n}{Q}$	$D = 8.68\sigma$	$D = \frac{8.68}{\tau}$
Logarithmic Decrement δ	$\delta = 2\pi\zeta$	$\delta = \frac{\pi\Delta\omega}{Q}$	$\delta = \frac{2\pi\sigma}{\omega_n}$	$\delta = \frac{2\pi}{\omega_n \tau}$

Table 1C. Damping Reference			
Parameter	Reverberation Time RT_{60} (sec)	Decay Rate D (dB/sec)	Logarithmic Decrement δ
3 dB Bandwidth $\Delta\omega$ (rad/sec)	$\Delta\omega = \frac{13.8}{RT_{60}}$	$\Delta\omega = \frac{D}{4.34}$	$\Delta\omega = \frac{\omega_n \delta}{\pi}$
3 dB Bandwidth Δf (Hz)	$\Delta f = \frac{2.2}{RT_{60}}$	$\Delta f = \frac{D}{27.3}$	$\Delta f = \frac{\omega_n \delta}{2\pi^2}$
Damping Frequency fd (Hz)	$fd = \frac{1.1}{RT_{60}}$	$fd = \frac{D}{54.5}$	$fd = \frac{\omega_n \delta}{4\pi^2}$
Loss Factor η	$\eta = \frac{13.8}{\omega_n RT_{60}}$	$\eta = \frac{D}{4.34\omega_n}$	$\eta = \frac{\delta}{\pi}$
Fraction of Critical Damping ζ	$\zeta = \frac{6.90}{\omega_n RT_{60}}$	$\zeta = \frac{D}{8.68\omega_n}$	$\zeta = \frac{\delta}{2\pi}$
Quality Factor Q	$Q = \frac{\omega_n RT_{60}}{13.8}$	$Q = \frac{4.34\omega_n}{D}$	$Q = \frac{\pi\omega_n}{\delta}$
Decay Constant σ (1/sec)	$\sigma = \frac{6.90}{RT_{60}}$	$\sigma = \frac{D}{8.68}$	$\sigma = \frac{\omega_n \delta}{2\pi}$
Time Constant τ (sec)	$\tau = \frac{RT_{60}}{6.90}$	$\tau = \frac{8.68}{D}$	$\tau = \frac{2\pi}{\omega_n \delta}$
Reverberation Time RT_{60} (sec)	–	$RT_{60} = \frac{60}{D}$	$RT_{60} = \frac{43.4}{\omega_n \delta}$
Decay Rate D (dB/sec)	$D = \frac{60}{RT_{60}}$	–	$D = 1.38\omega_n \delta$
Logarithmic Decrement δ	$\delta = \frac{43.4}{\omega_n RT_{60}}$	$\delta = \frac{\pi D}{4.34\omega_n}$	–

Reference

1. Svend Gade and Henrik Herlufsen, "Digital Filter versus FFT Techniques for Damping Measurement," Sound and Vibration, Bay Village, Ohio, March 1990.

DAMPING PROPERTIES OF MATERIALS

The purpose of this tutorial is to give typical damping values for various materials and systems.

The data in Tables 1 and 2 is taken from Reference 1.

Table 1.
Static Properties of Materials under Standard Conditions (approx. 20° C).

Material	Density (kg/m^3)	Elastic Modulus (N/m^2)	Shear Modulus (N/m^2)	Poisson's Ratio
Aluminum	2700	72 (10^9)	27 (10^9)	0.34
Lead	11,300	17 (10^9)	6 (10^9)	0.43
Iron	7800	200 (10^9)	77 (10^9)	0.30
Steel	7800	210 (10^9)	77 (10^9)	0.31
Gold	19,300	80 (10^9)	28 (10^9)	0.423
Copper	8900	125 (10^9)	46 (10^9)	0.35
Magnesium	1740	43 (10^9)	17 (10^9)	0.29
Brass	8500	95 (10^9)	36 (10^9)	0.33
Nickel	8900	205 (10^9)	77 (10^9)	0.30
Silver	10,500	80 (10^9)	29 (10^9)	0.37
Bismuth	9800	3.3 (10^9)	1.3 (10^9)	0.38
Zinc	7130	13.1(10^9)	5 (10^9)	0.33
Tin	7280	4.4 (10^9)	1.6 (10^9)	0.39

Table 2.
Dynamic Properties of Materials under Standard Conditions (approx. 20° C)

Material	Propagation Velocity of Longitudinal Wave in a Rod (meters/sec)	Propagation Velocity of Torsional Wave (meters/sec)	Longitudinal Loss Factor	Flexural Loss Factor
Aluminum	5200	3100	0.3 to 10 (10^{-5})	$\approx 10^{-4}$
Lead (pure)	1250	730	5 to 30 (10^{-2})	$\approx 2(10^{-2})$
Lead (including antimony)			1 to 4 (10^{-3})	
Iron	5050	3100	1 to 4 (10^{-4})	2 to 6 (10^{-4})
Steel	5100	3100	0.2 to 3 (10^{-4})	
Gold	2000	1200	$\approx 3\,(10^{-4})$	
Copper (polycrystalline)	3700	2300	$\approx 2\,(10^{-3})$	$\approx 2(10^{-3})$
Copper (single crystal)			2 to 7 (10^{-4})	
Magnesium	5000	3100		$\approx 10^{-4}$
Brass	3200	2100	0.2 to 1 (10^{-3})	$< 10^{-3}$
Nickel	4800	2900		$< 10^{-3}$
Silver	2700	1600	$\approx 4\,(10^{-4})$	$< 3\,(10^{-3})$
Bismuth	580	360		$\approx 8\,(10^{-4})$
Zinc	1350	850		$\approx 3\,(10^{-4})$
Tin	780	470		$\approx 20\,(10^{-4})$

Notes:

1. Some loss factors are unavailable.
2. The relationship between the loss factor η and the viscous damping ratio ξ is: $\eta = 2\xi$.

The data in Table 3 is taken from Reference 2.

Table 3. Representative Damping Ratios	
System	Viscous Damping Ratio ξ
Metals (in elastic range)	<0.01
Continuous Metal Structures	0.02 to 0.04
Metal Structure with Joints	0.03 to 0.07
Aluminum / Steel Transmission Lines	≈ 0.0004
Small Diameter Piping Systems	0.01 to 0.02
Large Diameter Piping Systems	0.02 to 0.03
Auto Shock Absorbers	≈ 0.30
Rubber	≈ 0.05
Large Buildings during Earthquakes	0.01 to 0.05
Prestressed Concrete Structures	0.02 to 0.05
Reinforced Concrete Structures	0.04 to 0.07

The data in Tables 4 through 6 is taken from Reference 3.

Table 4. Material Damping Ratios (Bare Structure)	
System	Viscous Damping Ratio ξ
Reinforced Concrete	
Small Stress Intensity (uncracked)	0.007 to 0.010
Medium Stress Intensity (fully cracked)	0.010 to 0.040
High Stress Intensity (fully cracked but no yielding of reinforcement)	0.005 to 0.008
Prestressed Concrete (uncracked)	0.04 to 0.07
Partially Prestressed Concrete (slightly cracked)	0.008 to 0.012
Composite	0.002 to 0.003
Steel	0.001 to 0.002

C.1

Table 5. Footbridge Damping			
Construction Type	Viscous Damping Ratio ξ		
	Min.	Mean	Max.
Reinforced Concrete	0.008	0.013	0.020
Prestressed Concrete	0.005	0.010	0.017
Composite	0.003	0.006	-
Steel	0.002	0.004	-

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Table 6. Building Damping			
Construction Type	Viscous Damping Ratio ξ		
	Min.	Mean	Max.
Tall Buildings (h > ~100 m)			
Reinforced concrete	0.010	0.015	0.020
Steel	0.007	0.010	0.013
Buildings (h ~ 50 m)			
Reinforced concrete	0.020	0.025	0.030
Steel	0.015	0.020	0.025

3.1

The damping values in the tables should be used with caution. There are many types of damping, such as viscous, hysteresis, acoustic coupling, air pumping at joints, energy radiation to the soil, etc. Also, boundaries and bearings contribute damping.

Furthermore, structures have many modes. Each mode may have a unique damping value.

References

1. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.

2. V. Adams and A. Askenazi, Building Better Products with Finite Element Analysis, OnWord Press, Santa Fe, N.M., 1999.

3. H. Bachmann, et al., Vibration Problems in Structures, Birkhauser Verlag, Berlin, 1995.